Validity and robustness of system dynamics global models: the case of World3

STEEP (INRIA) – MATHILDE DU PLESSIX

17th June 2021
Model presentation

- World3 was ordered by the Club of Rome to answer this question:

  «How may the expanding global population and material economy interact with and adapt to the earth’s limited carrying capacity over the coming decades? »

  *Limits to growth (2004)* [1]

- Great success of the report
- Model criticized for its simplicity and approximations

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System dynamics model

A system dynamic model is composed of:
- Positive or negative feedback loops
- Stocks
- Input and output flows
- Other variables
Model outputs

State of the world

Standard of living
Synthesis of the structure
Objectives

- Provide objective tools to assess what the model developers have underlined about its characteristics, behavior and the conclusions they have drawn from it, through automated model analysis.
- Provide elements to justify - or not - the robustness of the model
- Understand the dynamics of the model
Existing works

- Sensitivity analysis when the report was published
- Vermeulen and de Jhong [2] proposed to vary certain parameters by 10% to qualitatively change the outputs of the model
- Thissen’s thesis [3] analyzing each sector at a time
- World3 used as a tool for comparing sensitivity analysis methods

Tools

- Sensitivity Analysis
  - Screening method: Morris Method
  - Global analysis method: Sobol Method

- Loops dominance analysis:
  - Behavioral method
  - Structural dominance analysis: Loop Eigenvalue Elasticity Analysis
Sensitivity Analysis

Morris Method [4] :

- One at a time Method
- Can’t detect between non-linear behavior or parameters interactions

Sobol’ analysis [5] [6] :

- Detects parameters interactions
- Needs a lot of model simulations

Results

Parameters sensitivity:

**Population**
- Desired family size (DCFSN)
- Life expectancy (LE)

**Capital**
- Average Life of Industrial Capital (ALIC)
- Fraction of Industrial Capital Allocated to Consumption (FIOAC)
- Industrial Capital Output Ratio (ICOR)

**Agriculture**
- Land Fraction Harvested (LFH)

**Pollution**
- Persistent Pollution Transmission Delay (PPTD)
Loop dominance analysis

Ford Method [7]:

Variable of interest behavior

Deactivating loops

Model structure

Loop Eigenvalue Elasticity Analysis Method[8], [9]:

Model structure

Eigenvalues and eigenvectors

Modes of behavior


Loops dominance analysis

Structural analysis method: Loop Eigenvalue Elasticity Analysis (LEEA) ([8], [9])

1. Identify a Shortest Independent Loops Set (SILS) [10]

2. Compute the gain of each loop at each time step:
   - Gain of the variable $v$ on the variable $u$:
     $$ g(v, u) = \frac{du}{dv} $$
   - Loop gain: multiplication of variables gains composing the loop
     $$ g(loop) = g(v, u) \cdot g(z, v) \cdot g(u, z) $$

3. Compute the eigenvalues
   - Compute the gain matrix
   - Each eigenvalue defines a behavior mode of the system

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>Real part</th>
<th>Imaginary part</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td>The system diverge from a fix point</td>
<td>Growing oscillations</td>
</tr>
<tr>
<td>Negative</td>
<td>The system converge to a fix point</td>
<td>Dumping oscillations</td>
</tr>
</tbody>
</table>

4. Loop influence: change in the eigenvalue due to a small change in the gain of the loop:
   $$ \mu = \frac{d\lambda}{dg} \cdot g $$

Example of the Capital–Resources sector
Example of the Capital – Resources sector
Example of the Capital–Resources sector

Diagram showing relationships between industrial capital, industrial output, service capital, population, industrial output per capita, and other economic indicators. The diagram includes arrows indicating the flow of resources and investment, as well as a graph depicting industrial capital over time with a peak around 2025.
Example of the Capital – Resources sector
Example of the Capital – Resources sector
Loop dominance analysis: results

Capital and resources:
- The loop modeling the growth of the industrial capital (IC -> IO -> ICIR) is responsible for the instability in the sector.
- The lack of resources forces to invest less in the capital and is responsible for the decline.

Population:
- Mortality is driven by the food available.
- Birth is driven by the level of life.

Agriculture:
- Food production is mainly driven by land degradation at long term.

Pollution:
- The instability is exogenous to the sector.
- The positive feedback loop starts to exist on the second half of the simulation.
Work in progress

- Loop dominance analysis on the whole model

- Try new modelling choice into the existing sectors
  - example: try to determine FIOAC according to socioeconomic parameters
Conclusions

- First elements to objectify the speeches held concerning the model
- Implementation of tools that do not require knowledge of the model to understand its dynamics
- The authors were right in their explanation of the dynamics of the model
Sensitivity analysis: Morris Method

One-At-a-Time (OAT) method: Morris method [4]

Elementary effect of parameter $i$ for the trajectory $k$:

$$EE_i^k = \frac{M(x_1^k, \ldots, x_{i-1}^k, x_i^k + \delta, x_{k-1}^k, \ldots, x_d^k) - M(x_1^k, \ldots, x_{i-1}^k, x_i^k, x_{k-1}^k, \ldots, x_d^k)}{\delta}$$

Setting up simulations:

- Random draw of $r$ points on a regular grid of dimension $m \times m$
- Creation of $r$ trajectories of $(m+1)$ points by increasing or decreasing one at a time each element in the vector of parameters

Cost: $r(m+1)$ simulations

Morris method

- The mean of the elementary effects of the parameter $i$:
  \[ \mu_i = \frac{1}{r} \sum_{k=1}^{r} EE_i^k \]

- The standard deviation of the elementary effects of parameter $i$:
  \[ \mu_i^* = \frac{1}{r} \sum_{k=1}^{r} |EE_i^k| \]

- The mean of the absolute elementary effects of the parameter $i$:
  \[ \sigma_i = \sqrt{\frac{1}{r-1} \sum_{k=1}^{r} (EE_i^k - \mu_i)^2} \]
Sobol Analysis

Global analysis method: Sobol’ analysis ([5], [6])

➢ Takes each parameter as a random variable

➢ First order index – represents the influence of the parameter alone on the output

\[ S_i = \frac{V(E[Y|X_i])}{V(Y)} \]

When \( E[Y|X_i] \) is low, it means that fixing \( X_i \) reduces \( Y \) variability so \( X_i \) is influent

➢ Total order index- takes in account all the interactions where \( X_i \) is implied

\[ S_i = 1 - \frac{V(E[Y|X_{(-i)}])}{V(Y)} \]

➢ If \( S_i^T - S_i \) is close to 0 then the parameter influate directly the output of the model studied

Sobol’ Analysis: Results

First order and total indices for POP

Sobol indices according to time for POP
Empirical data camparison

Comparison with empirical data and 4 scenarios [11]