Validity and robustness of system dynamics global models: the case of World3

STEEP (INRIA) – MATHILDE DU PLESSIX

17th June 2021

Model presentation

- World3 was ordered by the Club of Rome to answer this question:

«How may the expanding global population and material economy interact with and adapt to the earth's limited carrying capacity over the coming decades? »

Limits to growth (2004) [1]

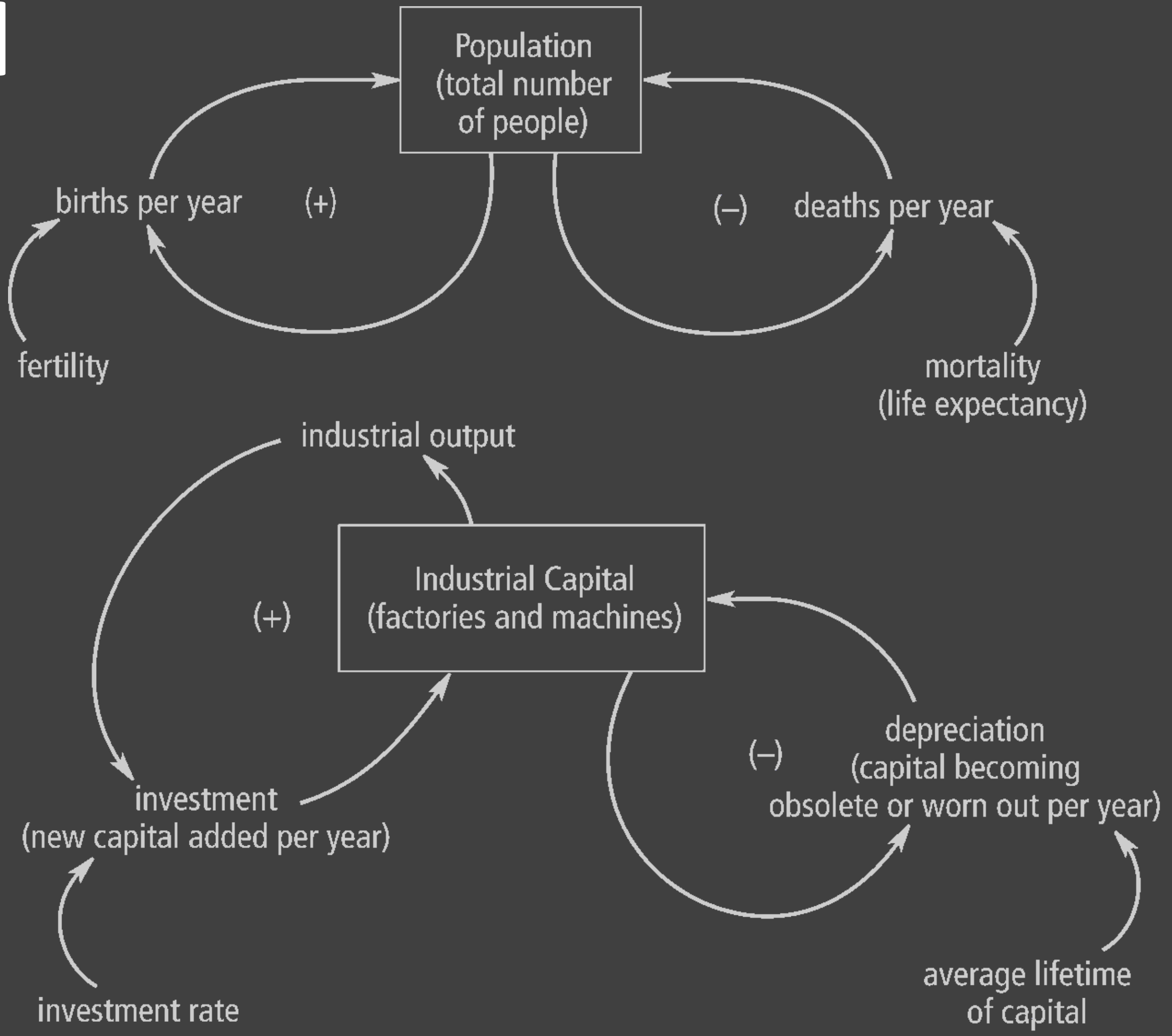
- Great success of the report
- Model criticized for its simplicity and approximations

[1] D. H. Meadows, J. Randers, et D. L. Meadows, *The Limits to Growth: The 30-year Update*, Rev Ed. London: Routledge, 2004.

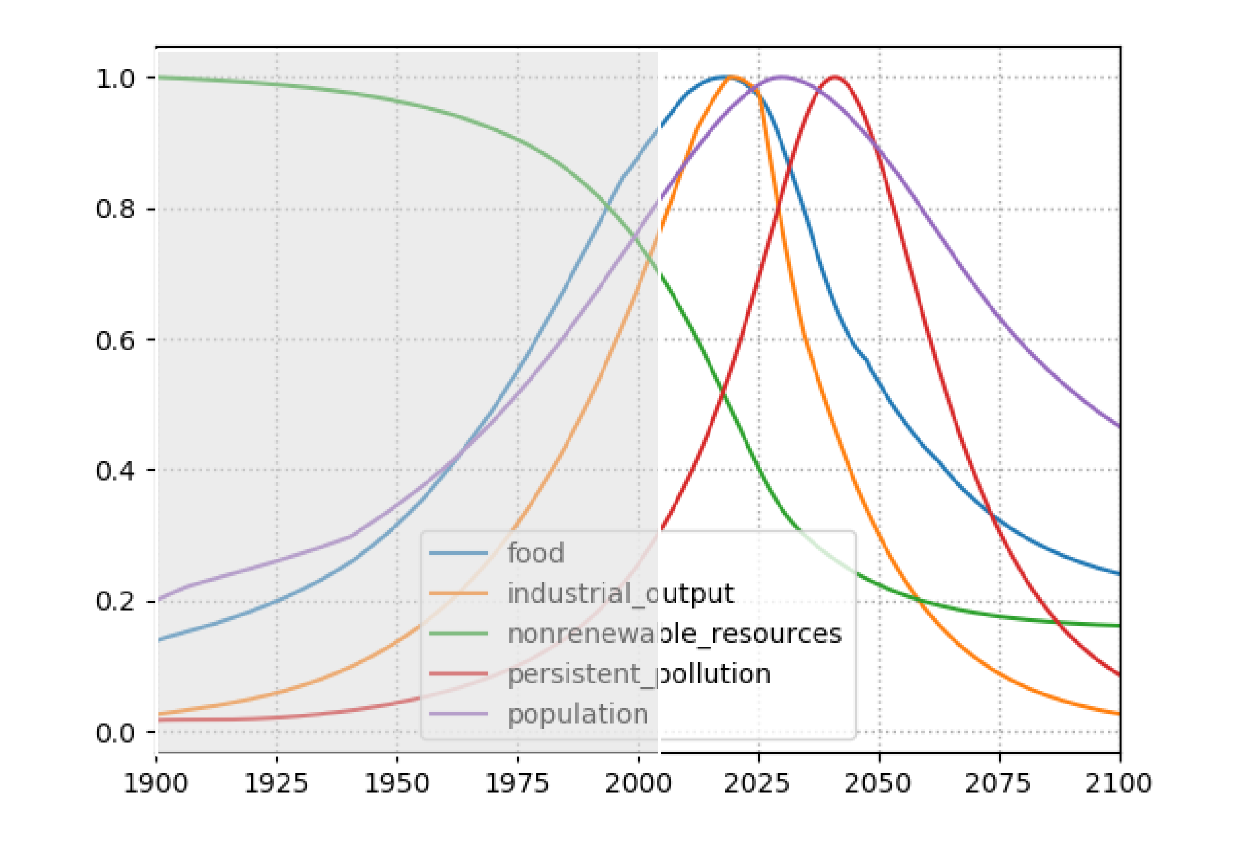
System dynamics model

A system dynamic model is composed of :

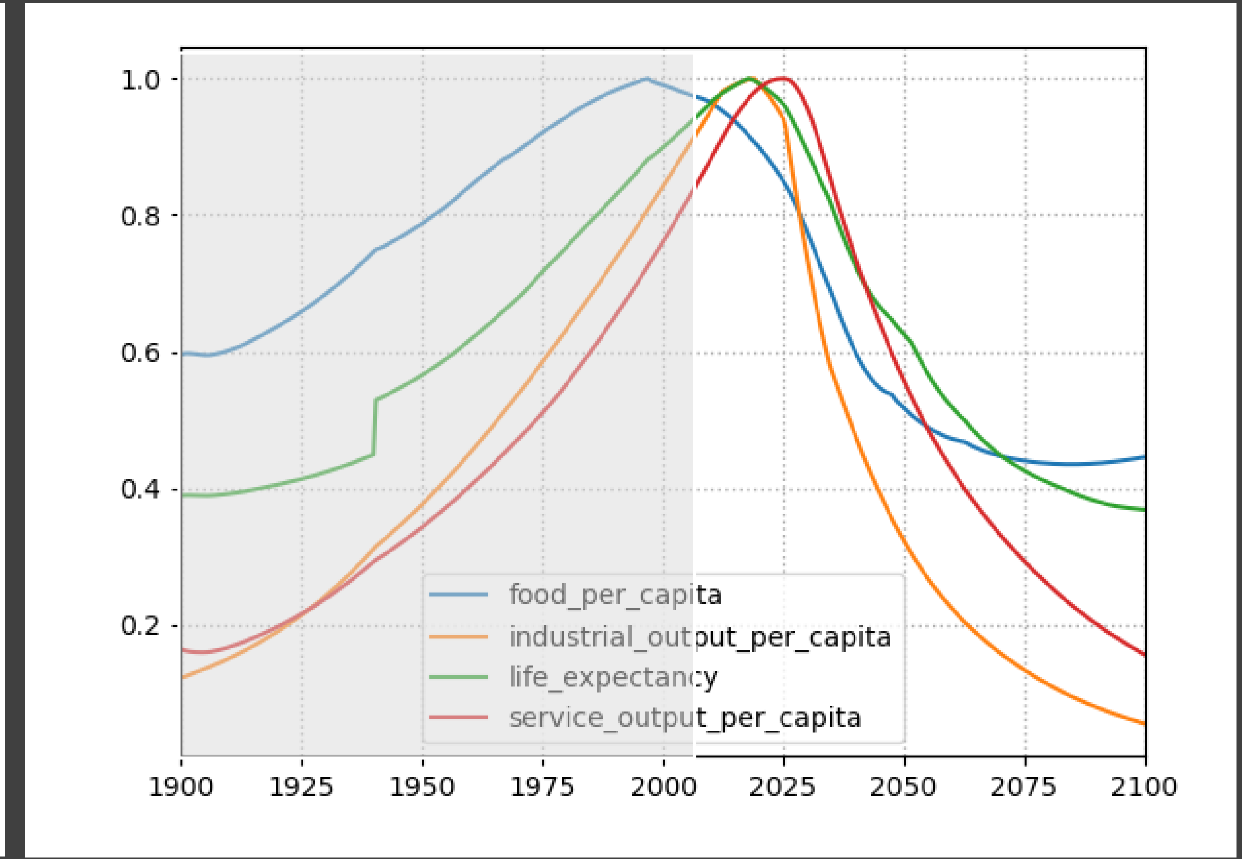
- Positive or negative feedback loops
- Stocks
- Input and output flows
- Other variables



Model outputs

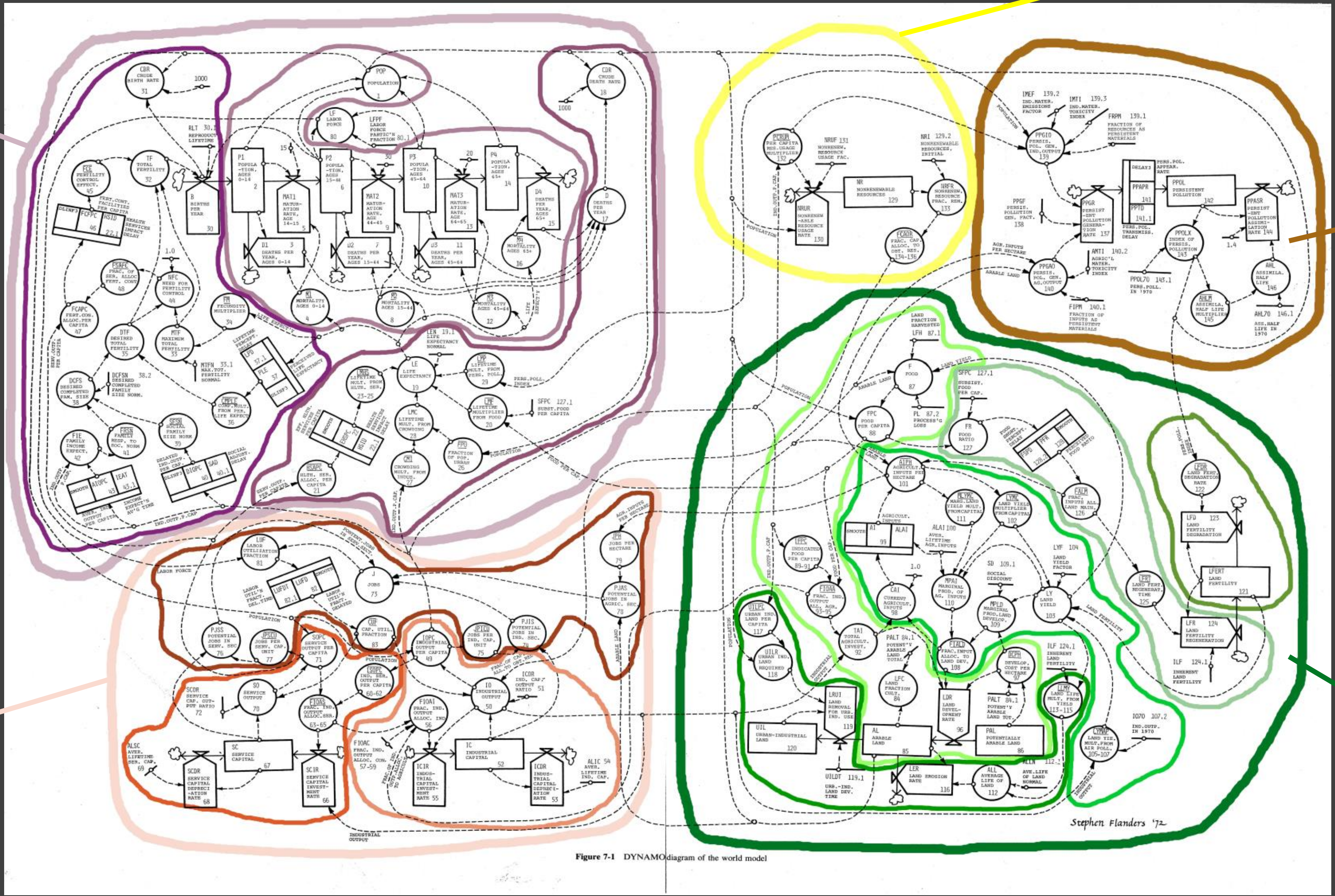


State of the world



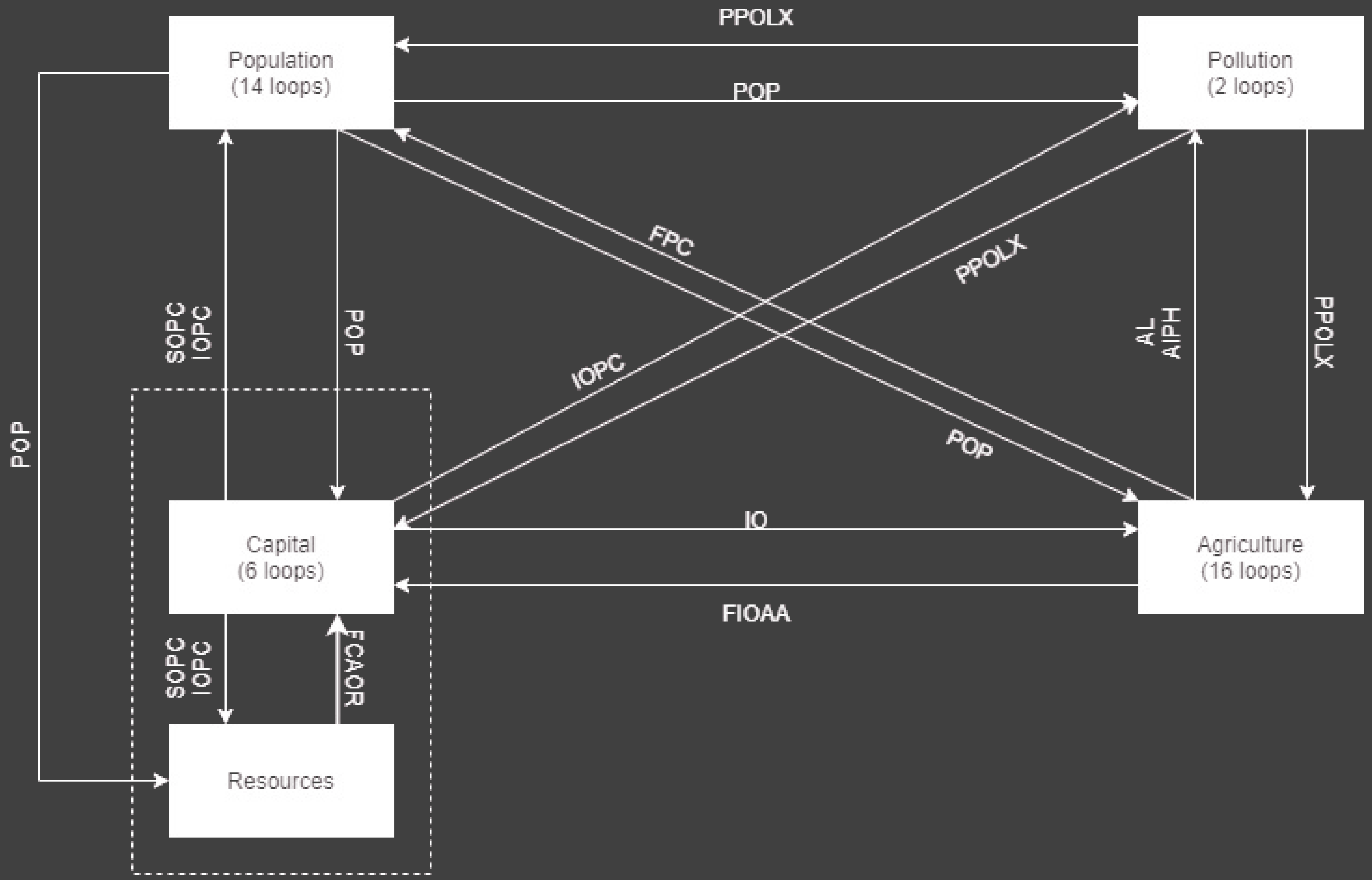
Standard of living

Model structure



DYNAMO Diagram (version of 1974)

Synthesis of the structure



Objectives

- Provide objective tools to assess what the model developers have underlined about its characteristics, behavior and the conclusions they have drawn from it, through automated model analysis.
- Provide elements to justify - or not - the robustness of the model
- Understand the dynamics of the model

Existing works

- Sensitivity analysis when the report was published
- Vermeulen and de Jhong [2] proposed to vary certain parameters by 10% to qualitatively change the outputs of the model
- Thissen’s thesis [3] analyzing each sector at a time
- World3 used as a tool for comparing sensitivity analysis methods

[2] P. J. Vermeulen and D. C. J. de Jongh, “Parameter sensitivity of the ‘Limits to Growth’ world model,” *Applied Mathematical Modelling*, vol. 1, no. 1, pp. 29–32, Jun. 1976
[3] W. Thissen, “Investigations into the World3 Model: The Capital and Resource Subsystem,” *IEEE Transactions on Systems, Man, and Cybernetics*, vol. SMC-6, no. 7, pp. 455–466, Jul. 1976.

Tools

- Sensitivity Analysis
  - Screening method : Morris Method
  - Global analysis method : Sobol Method

- Loops dominance analysis:
  - Behavioral method
  - Structural dominance analysis : Loop Eigenvalue Elasticity Analysis

Sensitivity Analysis

Morris Method [4] :

- One at a time Method
- Can't detect between non-linear behavior or parameters interactions

Sobol' analysis [5] [6]:

- Detects parameters interactions
- Needs a lot of model simulations

[4] M. D. Morris, « Factorial Sampling Plans for Preliminary Computational Experiments », *Technometrics*, vol. 33, nº 2, p. 161-174, mai 1991
[5] A. Saltelli, « Making best use of model evaluations to compute sensitivity indices », *Computer Physics Communications*, vol. 145, nº 2, p. 280-297, mai 2002
[6] M. Sobol', « Sensitivity Estimates for Nonlinear Mathematical Models », p. 8.

Results

Parameters sensitivity:

Population

- Desired family size(DCFSN)
- Life expectancy (LE)

Capital

- Average Life of Industrial Capital (ALIC)
- Fraction of Industrial Capital Allocated to Consumption (FIOAC)
- Industrial Capital Output Ratio (ICOR)

Agriculture

- Land Fraction Harvested (LFH)

Pollution

- Persistent Pollution Transmission Delay (PPTD)

Loop dominance analysis

Ford Method [7] :

Variable of interest behavior

↓

Deactivating loops

↓

Model structure

Loop Eigenvalue Elasticity Analysis Method[8], [9] :

Model structure

↓

Eigenvalues and eigenvectors

↓

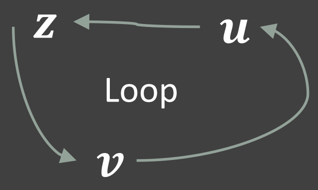
Modes of behavior

[7] D. N. Ford, « A behavioral approach to feedback loop dominance analysis », *System Dynamics Review*, vol. 15, nº 1, p. 3-36, 1999.
[8] C. E. Kampmann et R. Oliva, « Structural dominance analysis and theory building in system dynamics », *Systems Research and Behavioral Science*, vol. 25, nº 4, p. 505-519, 2008.
[9] C. E. Kampmann, « Feedback loop gains and system behavior (1996): Feedback Loop Gains », *Syst. Dyn. Rev.*, vol. 28, nº 4, p. 370-395, oct. 2012.

Loops dominance analysis

Structural analysis method: Loop Eigenvalue Elasticity Analysis (LEEA) ([8], [9])

1. Identify a Shortest Independent Loops Set (SILS) [10]



2. Compute the gain of each loop at each time step:
   - Gain of the variable v on the variable u :

$$\mathrm{g}(v,u) = \frac{du}{dv}$$

   - Loop gain= multiplication of variables gains composing the loop

$$\mathrm{g}(loop) = g(v,u) * g(z,v) * g(u,z)$$

3. Compute the eigenvalues
   - Compute the gain matrix
   - Each eigenvalue defines a behavior modes of the system

Eigenvalues	Real part	Imaginary part
Positive	The system diverge from a fix point	Growing oscillations
Negative	The system converge to a fix point	Dumping oscillations

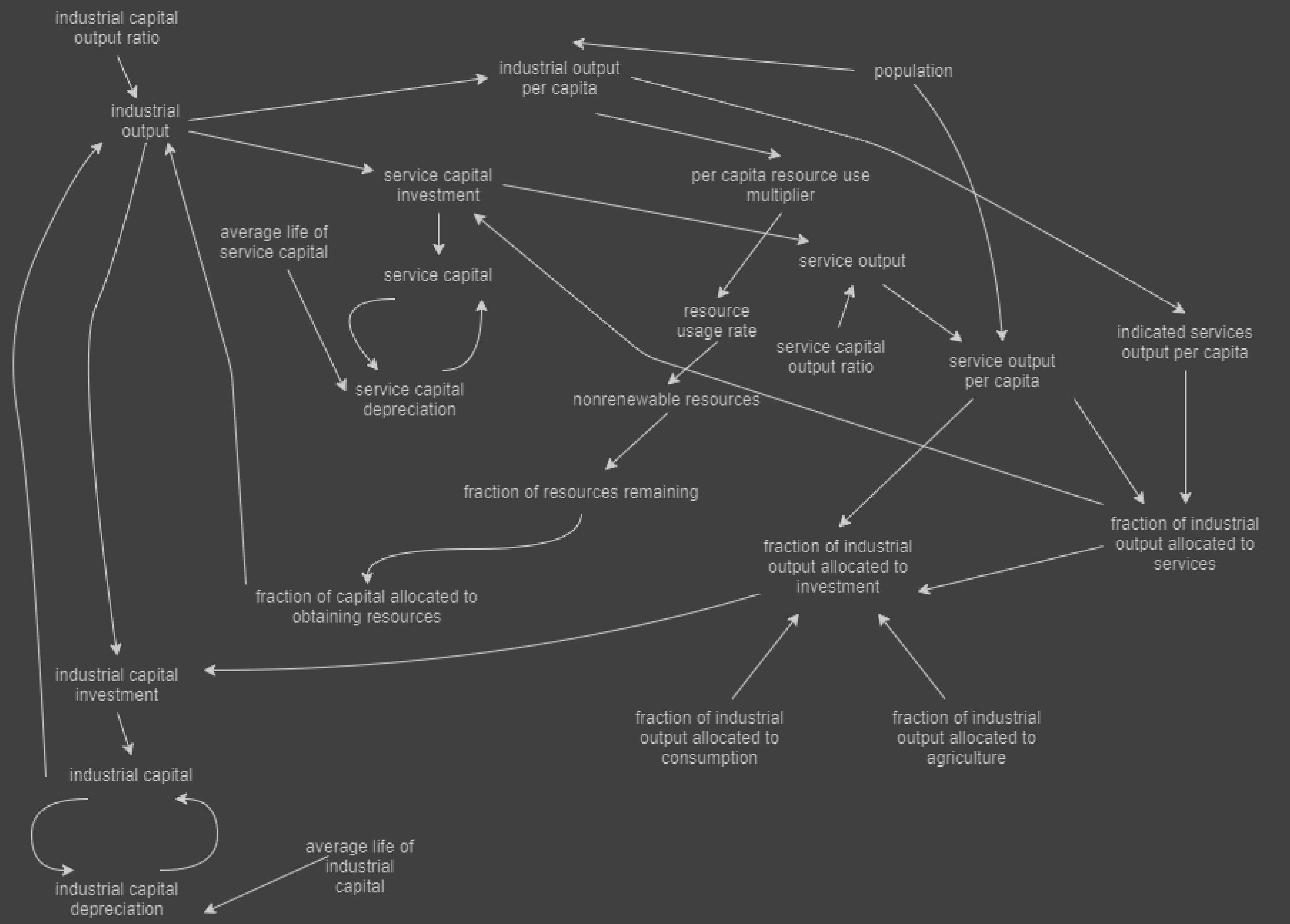
   - Determine which mode is dominant

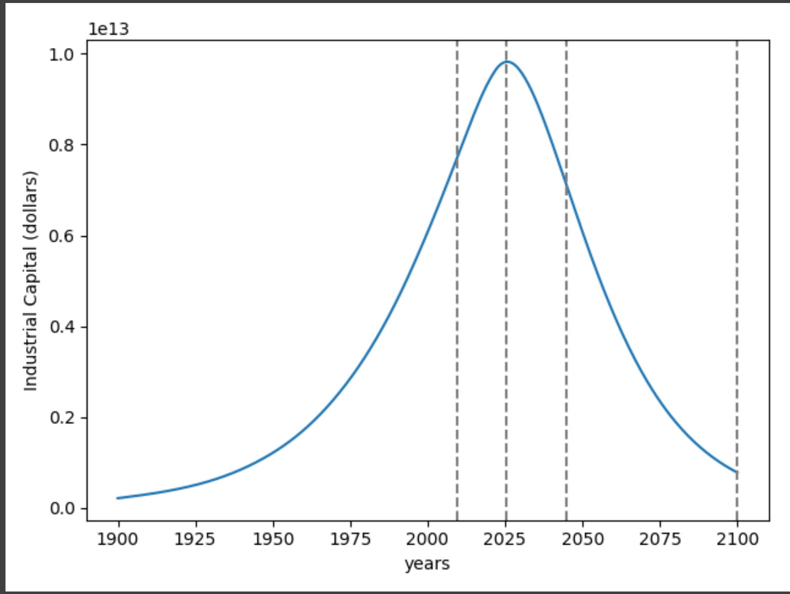
4. Loop influence: change in the eigenvalue due to a small change in the gain of the loop:

$$\mu = \frac{d\lambda}{dg} * g$$

[10] R. Oliva, « Model structure analysis through graph theory: partition heuristics and feedback structure decomposition », *Syst. Dyn. Rev.*, vol. 20, nº 4, p. 313-336, 2004.

Example of the Capital –Resources sector





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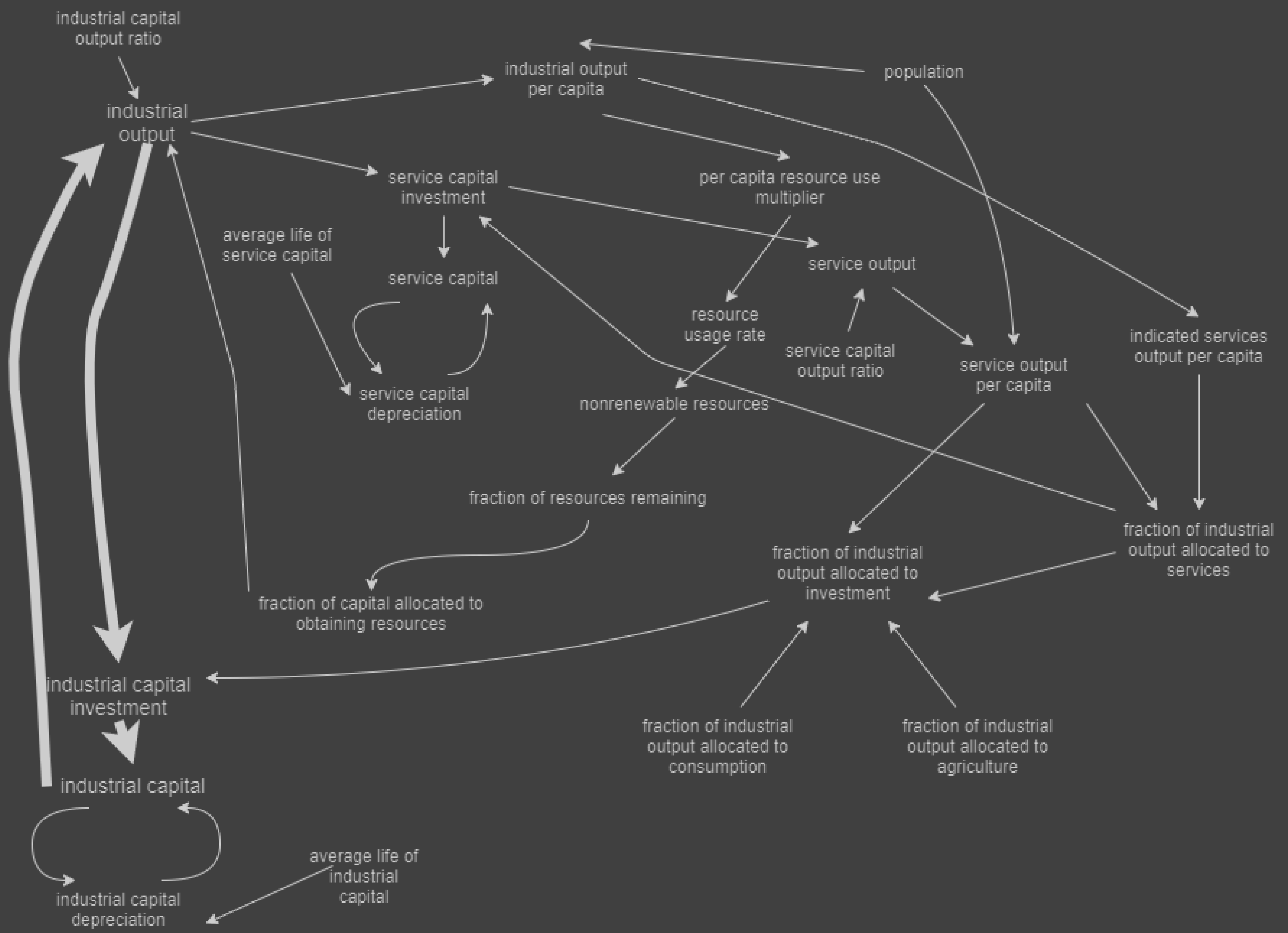
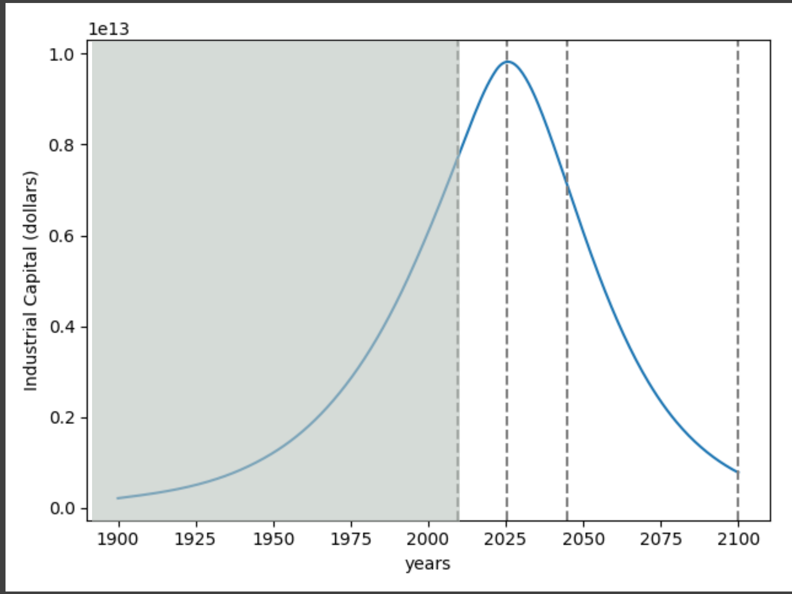
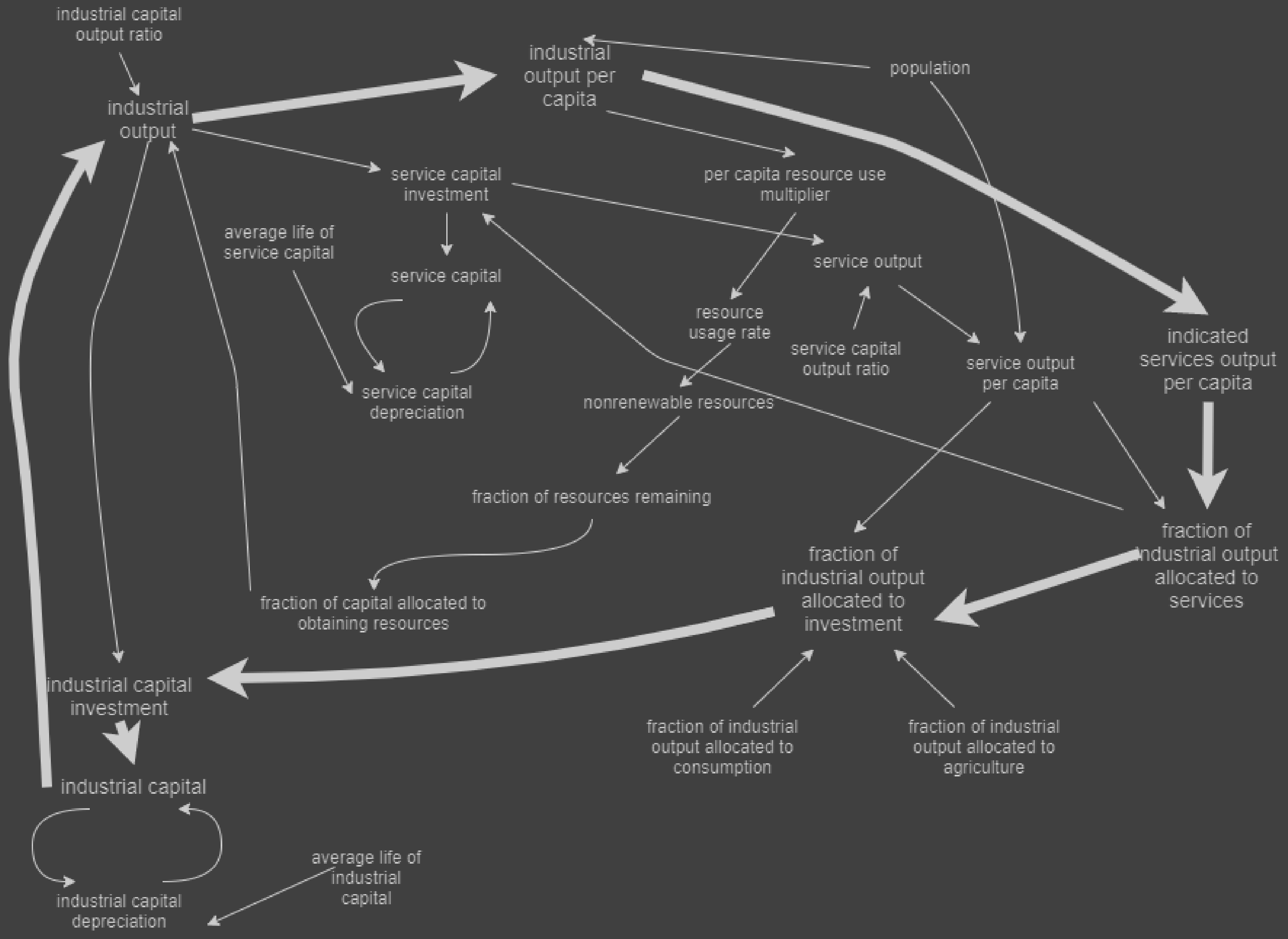
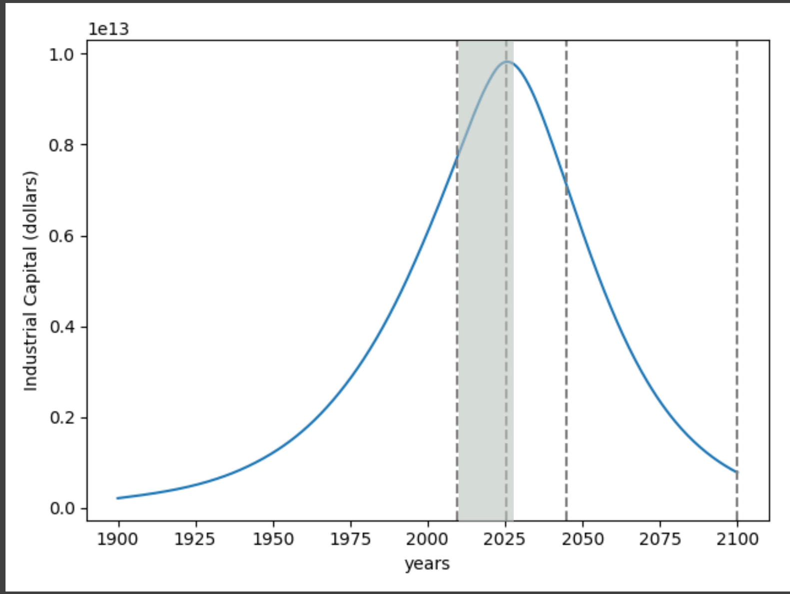




Example of the Capital –Resources sector





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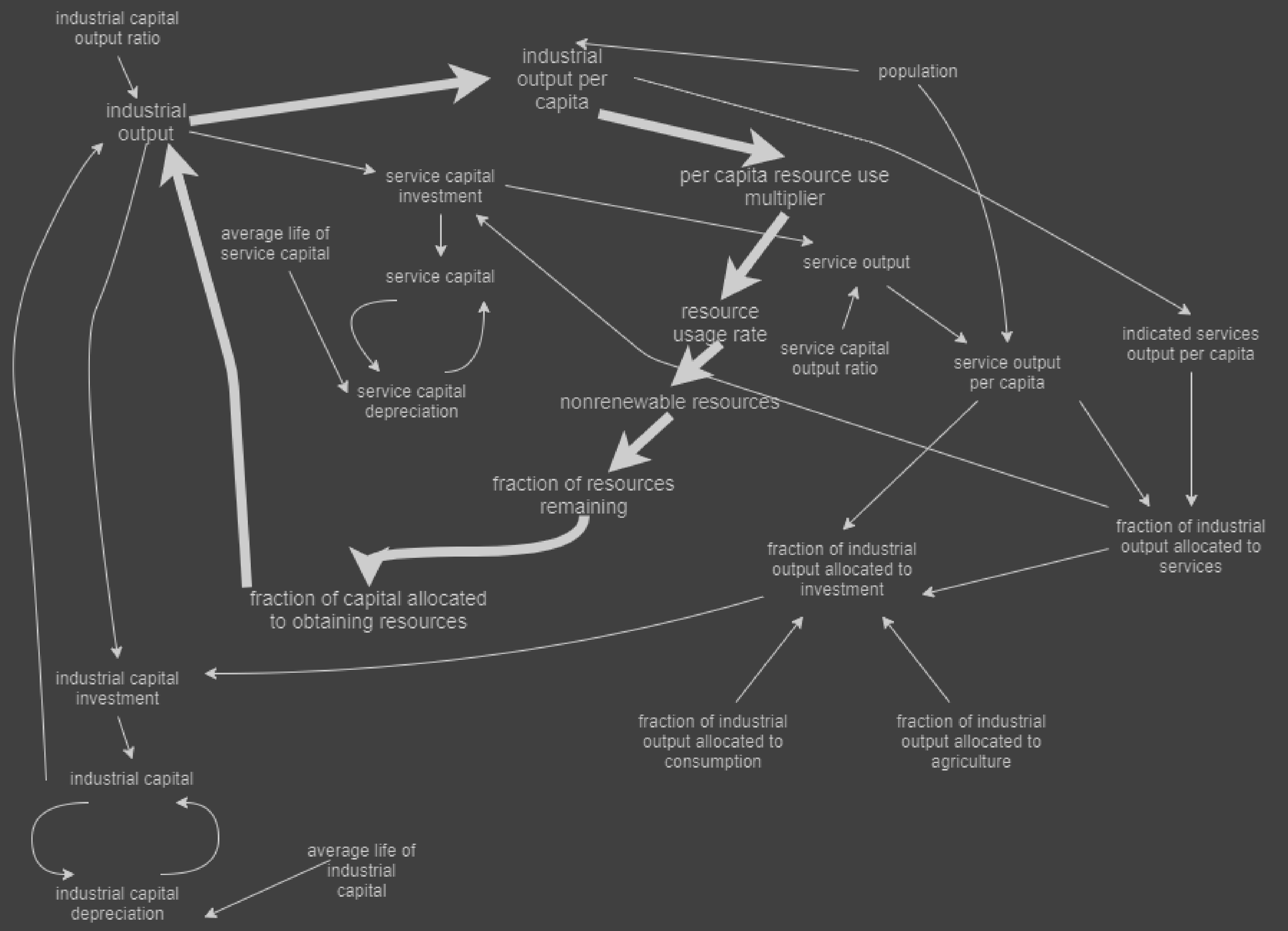

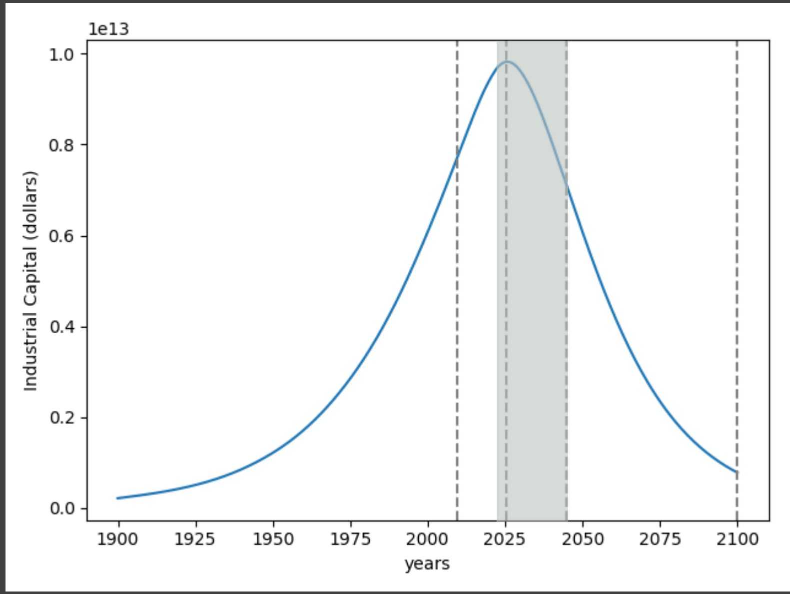

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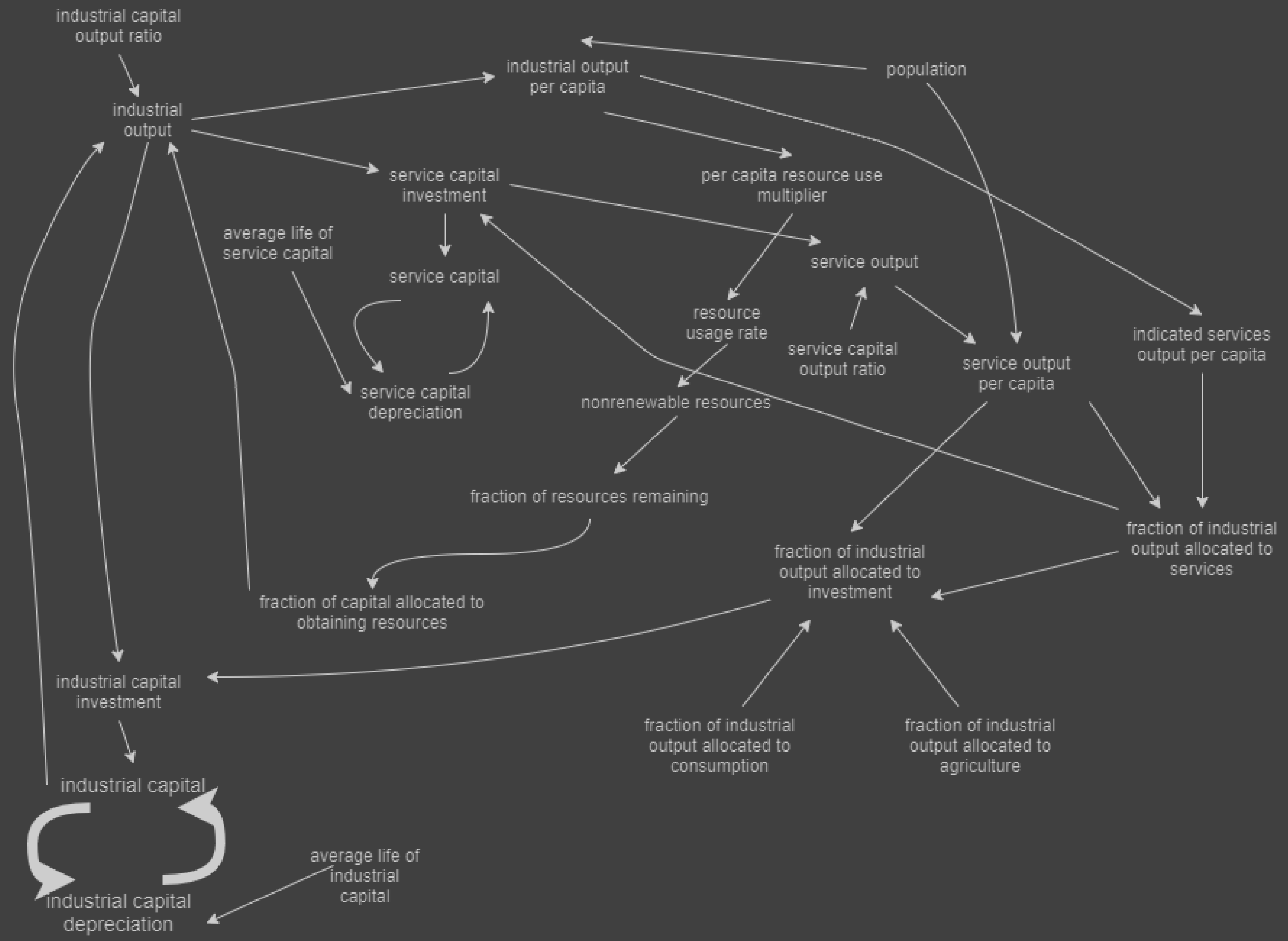
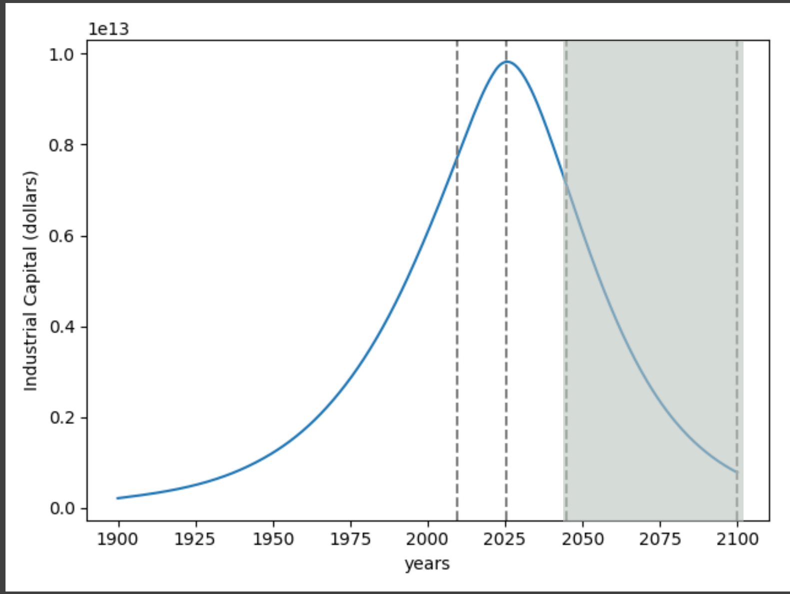




Loop dominance analysis : results

Capital and resources :

- The loop modeling the growth of the industrial capital (IC -> IO -> ICIR) is responsible for the instability in the sector
- The lack of resources forces to invest less in the capital and is responsible for the decline

Population :

- Mortality is driven by the food available
- Birth is driven by the level of life

Agriculture :

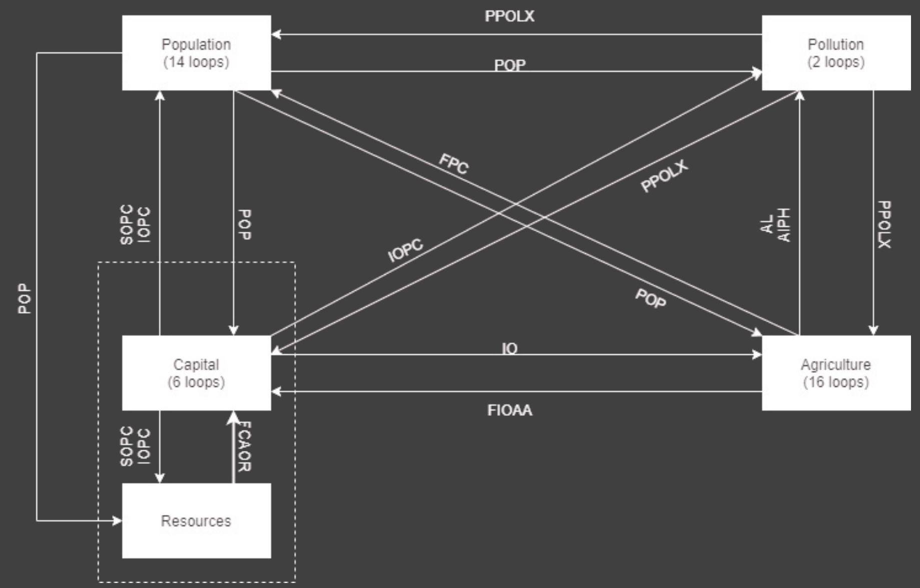
- Food production is mainly driven by land degradation at long term

Pollution

- The instability is exogenous to the sector
- The positive feedback loop starts to exist on the second half of the simulation

Work in progress

- Loop dominance analysis on the whole model



- Try new modelling choice into the existing sectors
  - example : try to determine FIOAC according to socioeconomic parameters

Conclusions

- First elements to objectify the speeches held concerning the model
- Implementation of tools that do not require knowledge of the model to understand its dynamics
- The authors were right in their explanation of the dynamics of the model

Sensitivity analysis: Morris Method

One-At-a-Time (OAT) method : Morris method [4]

Elementary effect of parameter i for the trajectory k:

$$EE_i^k = \frac{M\left(x_1^k, \dots, x_{i-1}^k, x_i^k+\delta, x_{k-1}^k, \dots, x_d^1\right)-M\left(x_1^k, \dots, x_{i-1}^k, x_i^k, x_{k-1}^k, \dots, x_d^1\right)}{\delta}$$

Setting up simulations:

- Random draw of r points on a regular grid of dimension m x m
- Creation of r trajectories of (m+1) points by increasing or decreasing one at a time each element in the vector of parameters

Cost: r(m+1) simulations

[4] M. D. Morris, « Factorial Sampling Plans for Preliminary Computational Experiments », *Technometrics*, vol. 33, n° 2, p. 161-174, mai 1991

Morris method

- The mean of the elementary effects of the parameter i:

$$\mu_i = \frac{1}{r}\sum_{k=1}^{r} EE_i^k$$

- The standard deviation of the elementary effects of parameter i:

$$\mu_i^* = \frac{1}{r}\sum_{k=1}^{r} |EE_i^k|$$

- The mean of the absolute elementary effects of the parameter i:

$$\sigma_i = \sqrt{\frac{1}{r-1}\sum_{k=1}^{r}(EE_i^k - \mu_i)^2}$$

	σ_i low	σ_i high
μ_i^* low	Negligible parameter	Influent parameter, non-linear effect and/or interactions
μ_i^* high	Influent parameter, linear effect	Influent parameter, non-linear effect and/or interactions

Sobol Analysis

Global analysis method: Sobol' analysis([5], [6])

- Takes each parameter as a random variable
- First order index – represents the influence of the parameter alone on the output

$$S_i = \frac{V(E[Y|X_i])}{V(Y)}$$

When $\mathrm{E}[Y|X_i]$ is low, it means that fixing X_i reduces Y variability so X_i is influent

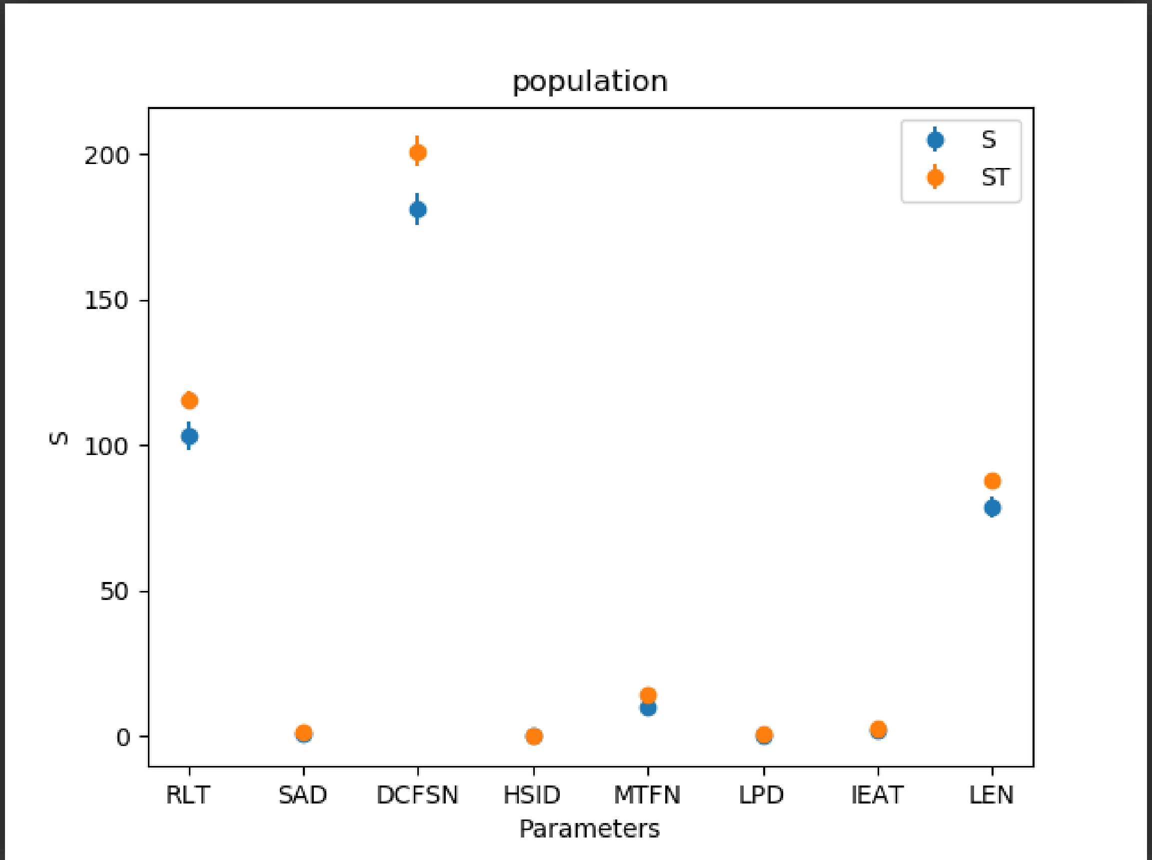
- Total order index- takes in account all the interactions where X_i is implied

$$S_i = 1 - \frac{V(E[Y|X_{(-i)}])}{V(Y)}$$

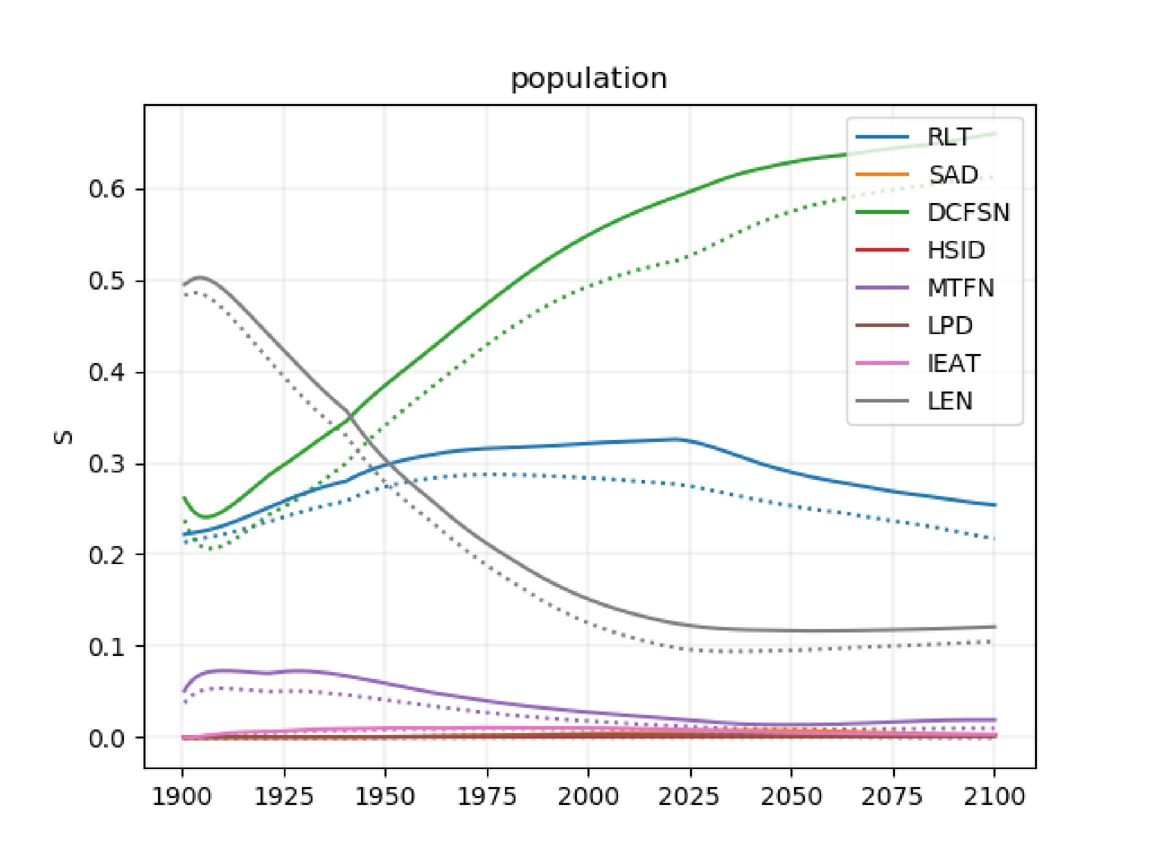
- If $S_i^T - S_i$ is close to 0 then the parameter influate directly the output of the model studied

[5] A. Saltelli, « Making best use of model evaluations to compute sensitivity indices », *Computer Physics Communications*, vol. 145, nº 2, p. 280-297, mai 2002
[6] M. Sobol', « Sensitivity Estimates for Nonlinear Mathematical Models », p. 8.

Sobol’ Analysis: Results

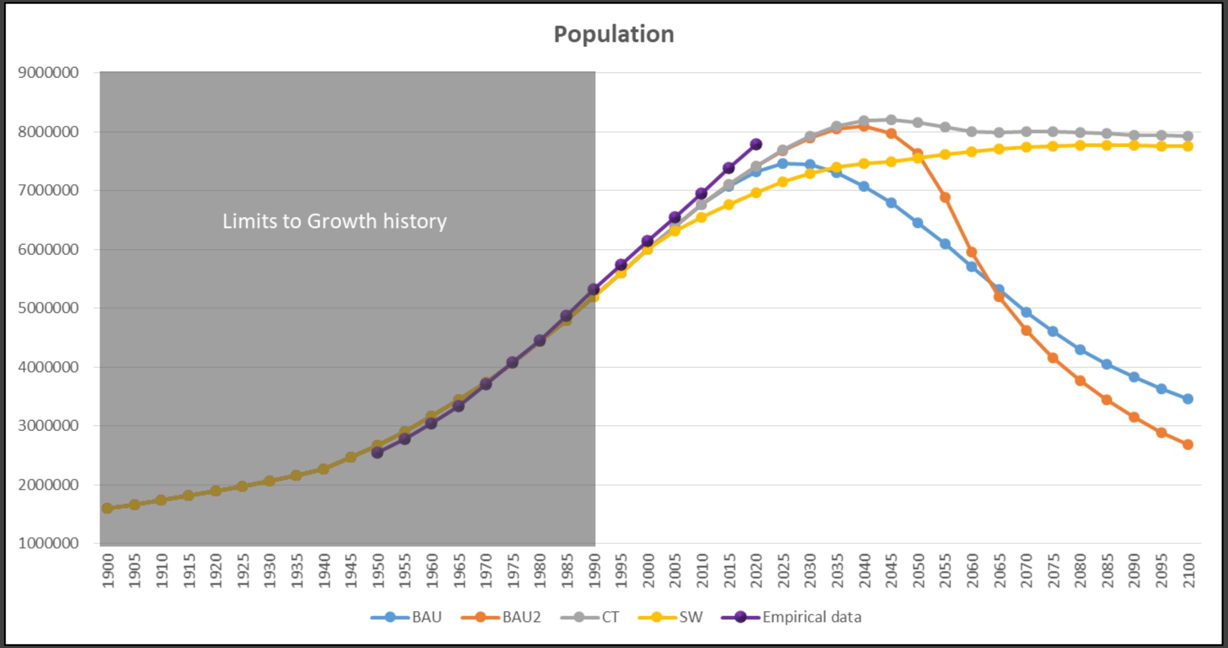


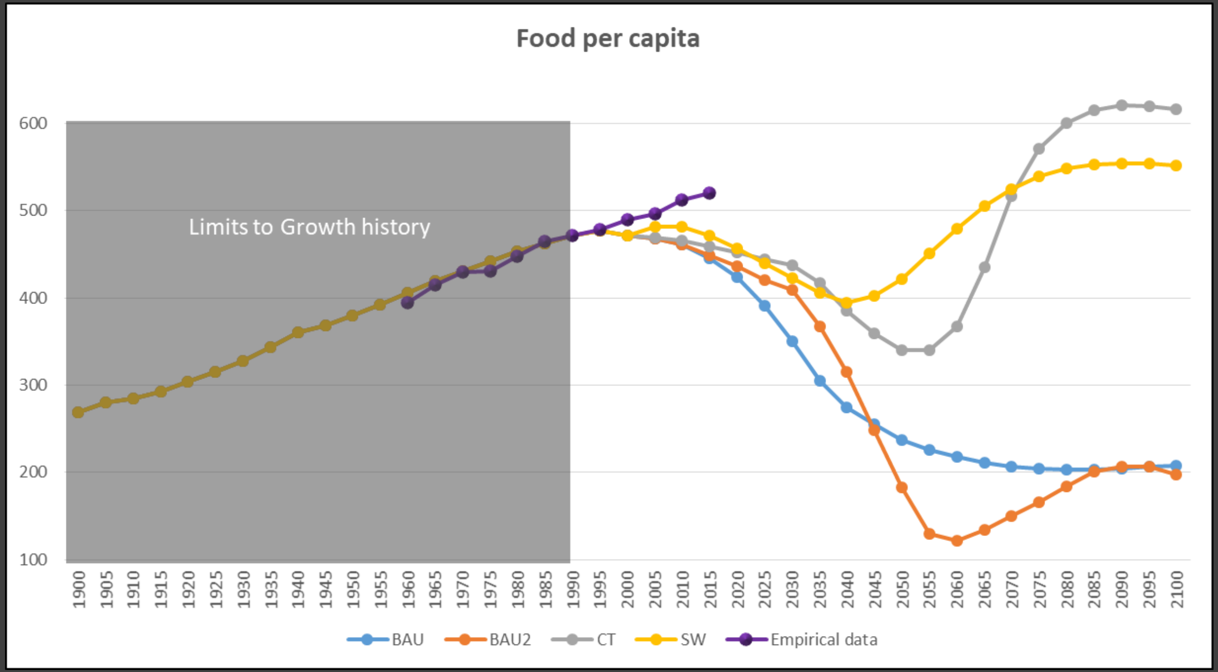
First order and total indices for POP



Sobol indices according to time for POP

Empirical data camparison





Comparison with empirical data and 4 scenarios [11]

[11] Herrington G. Update to limits to growth: Comparing the world3 model with empirical data. J Ind Ecol. 2020;1–13.